

# COUNTING MUONS TO PROBE THE NEUTRINO MASS SPECTRUM

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**ABSTRACT.** Neutrino oscillations depend on the neutrino mass spectrum. The expected difference between normal and inverted hierarchy is about 30% for muon neutrinos of 6-8 GeV that propagate for 6000-8000 km in the Earth. These requirements point to very long baseline neutrino experiments from CERN to Baikal Lake or from Fermilab to KM3NET (or to ANTARES) with conventional pion beams. Collecting  $10^{20}$  protons on target, one finds about a thousand events in a detector of 1 Mton, which makes it possible to observe the desired difference. The signal events are well characterized experimentally by their time and direction of arrival, and 2/3 of them are in a region with little atmospheric neutrino background, namely, between 4 GeV and 10 GeV. This can guarantee the identification of the neutrino mass spectrum just counting muon tracks.

## 1. INTRODUCTION

The study of neutrinos from the sun [1], from the Earth's atmosphere [2] and from artificial sources [3] led to discover neutrino oscillations [4] proving the relevance of the matter (MSW) effect [5] for solar neutrinos. The most direct extension of the standard model requires 3 massive neutrinos. Such a picture, summarized in Fig. 1 following [6], permits us to account for most neutrino observations and to ask new questions; in particular, how to probe whether the neutrino spectrum is normal or inverted. After the experimental evidence that  $\theta_{13}$  is large [7] a more precise formulation of this question is: how to take advantage of the matter effect in the Earth to solve this ambiguity.

The most relevant energies are indicated directly by the theory: the matter effect is maximum when the matter and the vacuum term that describe oscillations are comparable, and this happens when the neutrino energies are in the range 5 – 10 GeV. Moreover, neutrinos have to cross a sizeable amount of matter, of the order of the Earth's radius, as discussed later on (Sect. 2 and appendix A) and in agreement with the 'oscillograms' of [8] or with Fig. 3 of [9].

However, it is not easy to collect an adequate experimental sample of events where the matter effect is large [10]. An idea is to use atmospheric neutrinos, and more precisely, muon neutrinos and antineutrinos, because the easiest particles to detect are the muons and the antimuons. Several specific options have been considered, however all of them require a very large detector mass, due to the strong decrease of the atmospheric neutrino

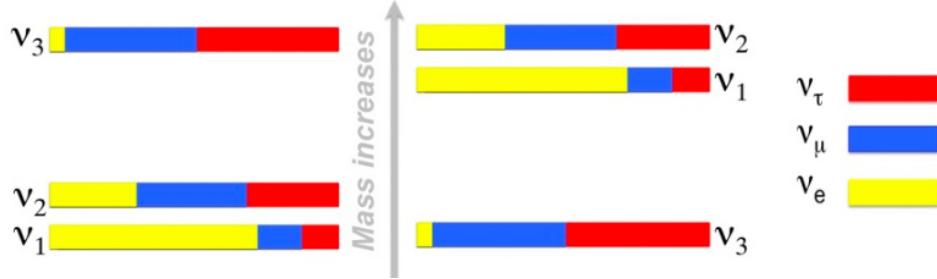


FIGURE 1. The two spectra that are compatible with present neutrino data: left, normal hierarchy; right, inverted hierarchy. The flavor content of each individual state with given mass,  $|U_{\ell i}^2|$ , is represented using a color code.

flux with energy.<sup>1</sup> Moreover, it is not possible to reconstruct the neutrino energies and directions simply observing the muons; in fact, a muon neutrino with energy  $E$  will give a muon with energies from zero to this value, and the angle between the neutrino and the produced muon is  $\sim \sqrt{1 \text{ GeV}/E} \sim$  several tens of degrees.

With these considerations in mind, we would like to suggest a different approach to emphasize the physics effect we are interested in, insisting on the use of large muon detectors. We propose to send a muon neutrino beam produced in laboratory, with energies and at a distance chosen to maximize the matter effect. In other terms, we suggest that in order to distinguish normal from inverted hierarchy, we should simply count muons in the right type of experimental setup.

## 2. MUON SURVIVAL PROBABILITY

We begin from the case when only the largest  $\Delta m^2$  matters. If  $\theta_{13}$  is set to zero, we have 2-flavor vacuum oscillation: the muon survival probability oscillates as a function of the energy, its maxima correspond to the minima of  $P_{\mu\tau}$  (and viceversa), and there is no difference between normal and inverted hierarchy. Then, let us consider the effect of  $\theta_{13}$ . Assuming normal hierarchy,  $P_{\mu e}$  is amplified due to matter effect for certain energies. Therefore, the muon survival probability,

$$(1) \quad P_{\mu\mu} = 1 - P_{\mu e} - P_{\mu\tau}$$

must decrease. Instead, for inverted hierarchy  $P_{\mu e}$  is suppressed by matter effect and the oscillations are very similar to the case when  $\theta_{13}$  is set to zero.

Being interested in maximizing the difference between normal and inverted hierarchy, we consider the case when a *local maximum* of  $P_{\mu\mu}$  is decreased as much as possible. This happens when the first minimum of  $P_{\mu\tau}$ , that drives the maximum of  $P_{\mu\mu}$ , falls close to

<sup>1</sup>E.g., a 50 kton magnetized iron detector, able to measure the charge of the muon [11] and to distinguish between neutrinos and antineutrinos; many 100 kton argon detectors, able to measure also the energy of the hadrons scattered by the neutrinos, reconstructing better the neutrino energy [12]; a huge,  $\sim 10$  Mton water Cherenkov muon detector [8], as a dense core of underwater/underice installations aimed at seeing high-energy neutrinos from cosmic sources.

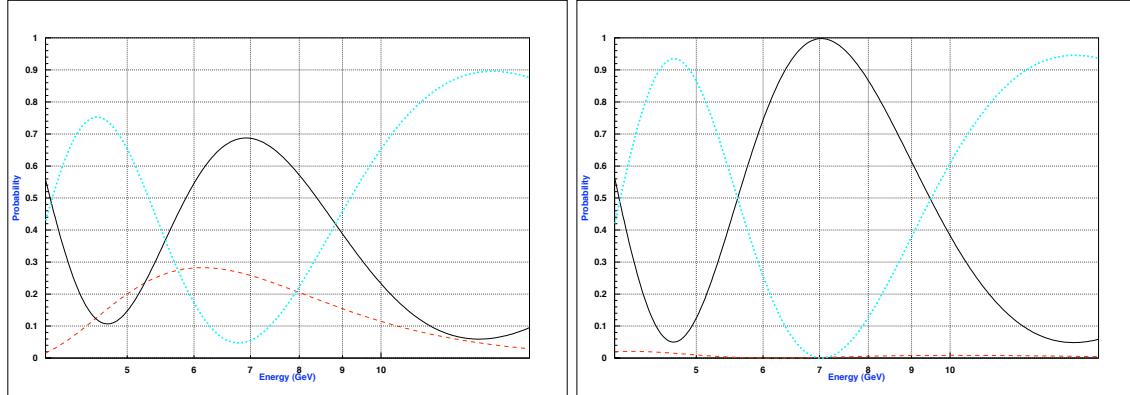


FIGURE 2. Oscillation probabilities for  $L = 7000$  km: the continuous (black) line is  $P_{\mu\mu}$ ; the dashed (red) line  $P_{\mu e}$ ; the dotted (cyan) line  $P_{\mu\tau}$ . Left/right panel, normal/inverted hierarchy. The increase of  $P_{\mu e}$  for normal hierarchy, due to matter effect, causes the decrease of  $P_{\mu\mu}$ .

the energy where  $P_{\mu e}$  is large. Such a condition can be analyzed easily and efficiently using the web resource,

<http://pcbat1.mi.infn.it/~battist/cgi-bin/oscil/index.r>

A FORTRAN code, developed for the study [12] where it is described, integrates numerically the hamiltonian for neutrino oscillations in the Earth with an accuracy of better than 1 part per million. Let us adopt the result of the global analysis of the Bari group [13], namely,  $\theta_{12} = 33.6^\circ$ ,  $\theta_{13} = 8.9/9.0^\circ$ ,  $\theta_{23} = 38.4/38.8^\circ$ ,  $\delta = 194/196^\circ$ ,  $\Delta m_{12}^2 = 7.54 \times 10^{-5}$  eV $^2$ ,  $\Delta m_{23}^2 = 2.39/2.47 \times 10^{-3}$  eV $^2$ ; where the two values apply to normal/inverted hierarchy, respectively. When the neutrino energy  $E$  and the corresponding oscillation length  $L$  are about

$$(2) \quad E = 6, 7, 8 \text{ GeV and } L = 6000, 7000, 8000 \text{ km respectively}$$

we have that

$$(3) \quad P_{\mu\mu}^{\max}(\text{NH}) \sim 0.7$$

Therefore, the difference with  $P_{\mu\mu}^{\max}(\text{IH}) \sim 1$ , in the case of inverted hierarchy, is as large as 30% for the distances given above,  $L \sim R_\oplus$ . As an example, we plot in Fig. 2 the case  $L = 7000$  km.

We verified that for other distances and/or other energies, the effect is smaller, especially when  $L$  decreases. Thus the above conditions identify the optimal distances and energies to search for matter effect on  $P_{\mu\mu}$ , and impose conditions on the type of muon neutrino source and muon detectors needed for this purpose. We checked that these results do not depend much on the errors on most of the parameters, and in particular on CP-violating phase. The largest variation is due to  $\theta_{23}$ ; for instance, the  $3\sigma$  range [13] implies that at 7800 km  $P_{\mu\mu}^{\max}(\text{NH}) \sim 0.7^{+0.04}_{-0.15}$ , that is not a very wide variation, and moreover, there are

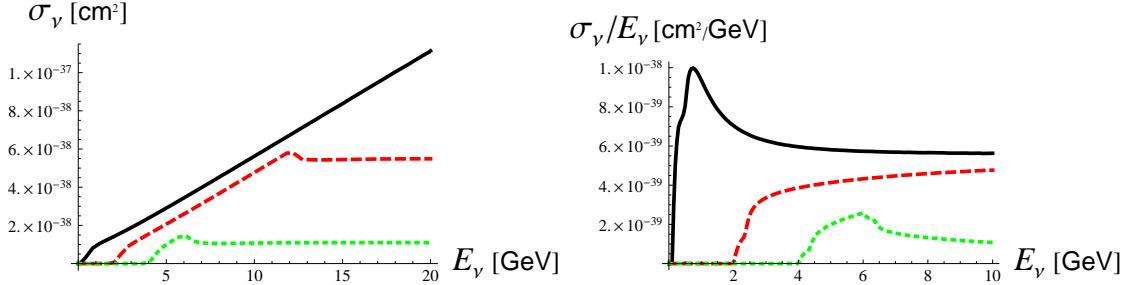


FIGURE 3. Various  $\nu_\mu N \rightarrow \mu X$  cross sections as a function of the energy of the incoming  $\nu_\mu$ . The continuous (black) line is the total cross section; the dashed (red) line restricts the muon energy to  $2 < E_\mu < 12$  GeV; the dotted (green) to  $4 < E_\mu < 6$  GeV.

reasonable perspectives that  $\theta_{23}$  will improve in the next few years. Appendix A offers further discussion of the matter effect enriched by analytical formulae (e.g., Eqs. 10 and 17 show that the effect increases with  $\sin^2 \theta_{23}$ ).

### 3. CROSS SECTION FOR NEUTRINO-MUON PRODUCTION

For the energies of interest, the leading interaction that yields observable muons is,

$$(4) \quad \nu_\mu + N \rightarrow \mu^- + X$$

where  $N$  is an average nucleon and  $X$  is a set of hadrons; we consider water nuclei, therefore a neutron/proton ratio of 4/5. We use the differential cross section  $d\sigma/dE_\mu$  calculated in [14], following [15]. In view of the fact that the neutrino energies are not very large, we sum the following three contributions according to the hadronic mass of the final state  $m_X^2 \equiv p_X^2$ :

- (1) The quasi-elastic contribution, with  $M_A^2 = 0.95$  GeV<sup>2</sup> and  $F_A(0) = -1.26$  [16].
- (2) The delta resonance, where we resort to CVC and PCAC and the parameterization of the form factors described in eqs. 12, 13 and 18 of [17].
- (3) The deep inelastic contribution for  $m_X > 1.4$  GeV, with GRV94 partons [18].

We show in Fig. 3 the following integrated cross sections: 1) the total cross section; 2) the one where we integrate only the region of muon energy  $2 < E_\mu < 12$  GeV; 3) the same, restricted to the range  $4 < E_\mu < 6$  GeV. As customary after [15], also the ratio between the cross section and the neutrino energy is given. At these energies, we find an approximate scaling of the integrated cross section:  $\sigma(E_\nu, E_1 < E_\mu < E_2) \equiv \int_{E_1}^{E_2} \frac{d\sigma}{dE_\mu}(E_\nu, E_\mu) dE_\mu \approx f(E_\nu - E_1, E_2 - E_1)$ . Note that when  $E_\nu$  is close to the threshold, the contribution of individual resonances considered (the nucleon and the delta) is clearly visible.

### 4. NEEDS FOR THE EXPERIMENT

**4.1. Mass of the detector.** In order to have a reasonable event counting rate, we note that the number of muon events scales with the mass of the detector  $M_{\text{det}}$  and the distance

$L$  as  $N_\mu \propto M_{\text{det}}/L^2$ . Thus, we have approximatively as many events in a detector of 10 kton at 800 km from the source (roughly corresponding to the present generation long baseline experiment NO $\nu$ A [19]) and in a detector of 1 Mton located at 8000 km. Moreover, we need to detect muons from few to ten GeV (see Fig. 2) which means that we have to measure tracks of 10 – 40 meters in water.

These requirements, concerning the mass of the detector and the length of the tracks to be revealed, point toward a large but relatively simple underwater or under-ice detector, that could resemble the PINGU [20] and ORCA [21] proposals that are being developed/considered by the IceCube and KM3NET collaborations, respectively. In the following, we will assume that the detector has a number of useful target nucleons of

$$(5) \quad N_{\text{targ.}} = 6 \times 10^{35}$$

corresponding to a cube of water of 100 m in size, i.e.,  $M_{\text{det}} = 1$  Mton, and leave a more detailed description of the detector for future work. Our results can be simply rescaled with the mass of the detector.

**4.2. Source-detector distance.** The next question is which arrangement of source and detector would fit the optimal distance identified above. Let us consider the existing neutrino lines [22] and a few sites of under-water and under-ice neutrino experiments. Their approximate coordinates in degrees are

$$(6) \quad (\lambda, \phi) = \begin{cases} (+41.8, -88.3) & \text{Fermilab} \\ (+46.2, +6.0) & \text{CERN} \\ (+36.4, +140.6) & \text{J-PARC} \end{cases} \quad \text{and} \quad = \begin{cases} (-90, +0.0) & \text{South Pole} \\ (+36.3, +16.1) & \text{Sicily} \\ (+51.8, +104.3) & \text{Baikal Lake} \end{cases}$$

‘South Pole’, ‘Sicily’ and ‘Baikal Lake’ correspond to the coordinates of IceCube, KM3NET and GVD, respectively. From the position versors  $\vec{n}_i = (\cos \lambda_i \cos \phi_i, \cos \lambda_i \sin \phi_i, \sin \lambda_i)$  we find the relative distances as  $L = R_\oplus |\vec{n}_1 - \vec{n}_2| = R_\oplus \sqrt{2(1 - \vec{n}_1 \cdot \vec{n}_2)}$ ; or, using the web resource [23] one gets the minimum distance on the Earth surface  $A$  (the arc) and thus  $L = 2R_\oplus \sin[A/(2R_\oplus)]$ . We find the following distances, expressed in km

	Fermilab	CERN	J-PARC
South Pole	11600	11800	11400
Sicily	<b>7800</b>	1230	9100
Baikal Lake	8700	<b>6300</b>	3300

The pairs CERN to Baikal Lake and Fermilab to Sicily are at an optimal distances. The first case would be somewhat more convenient, for its  $1/L^2$  factor is 50% larger. However, as we will show, also the second case offers reasonable opportunities for an experiment, and before continuing, we note that also the location of ANTARES, with  $(\lambda, \phi) = (+42.8, +6.2)$ , would be favourable, its distance from Fermilab being 6900 km.

**4.3. Properties of the neutrino beam.** The existing neutrino beam from CERN has a mean energy twice than necessary to probe the matter effect, whereas it has already been shown that the NuMI neutrino beam can be arranged to fit our needs [24]. The estimated fluences, and more precisely those called HE and ME options [24], are evaluated for  $10^{20}$  protons on target and at 1.04 km from the source. They can be reasonably approximated as

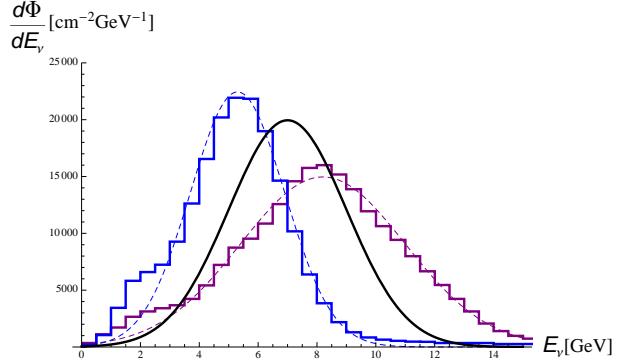


FIGURE 4. Differential neutrino fluences for a distance of  $L = 7800$  km. The two dashed gaussian distributions at the right and at the left are quite similar to the fluxes ME and HE estimated in [24] for  $10^{20}$  protons-on-target, shown as histograms. The black gaussian distribution with intermediate energies is assumed for the subsequent calculations. Compare with Fig. 2.

gaussian distributions, proportional to  $G(E, \bar{E}, \delta E) = \exp[-(E - \bar{E})^2/(2\delta E^2)]/\sqrt{2\pi \delta E^2}$ , with  $\bar{E} = 8.2$  (resp., 5.3) GeV and  $\delta E = 2.8$  (resp., 1.6) GeV as shown in Fig. 4, where we rescaled for a distance of  $L = 7800$  km. The fluence of a conventional muon neutrino beam, with properties intermediate between the above two cases (see Fig. 4), again corresponding to  $10^{20}$  protons on target and at the same distance, is approximated by

$$(7) \quad \frac{d\Phi}{dE} = \frac{10^5 \nu_\mu}{\text{cm}^2} \times G(E, \bar{E}, \delta E) \quad \text{with} \quad \begin{cases} \bar{E} = 7 \text{ GeV} \\ \delta E = 2 \text{ GeV} \end{cases}$$

In the following, we will consider such a fluence for definiteness.

## 5. RESULTS

The number of expected muon tracks contained in the detector, corresponding to muon energies larger than  $E_{th}$  and to a given fluence of muon neutrinos  $d\Phi/dE$ , is

$$(8) \quad N_\mu = N_{\text{targ.}} \times \int_{E_{th}} P_{\mu\mu}(E) \times \frac{d\Phi}{dE}(E) \times \sigma(E) \, dE$$

When we use the number of targets as in Eq. 5, the fluence of Eq. 7 (see previous section) and the cross section to produce muons above 2 GeV and below 12 GeV (Sect. 3) we get

$$(9) \quad N_\mu = \begin{cases} 950 \text{ with normal hierarchy} \\ 1300 \text{ with inverted hierarchy} \end{cases}$$

the difference is 30%, as expected. The distribution of the events is shown in Fig. 5. Even if we retain only the events with muon energy above 4 GeV, in order to avoid confusion with atmospheric neutrinos, we loose only 1/3 of the events. The number of signals is

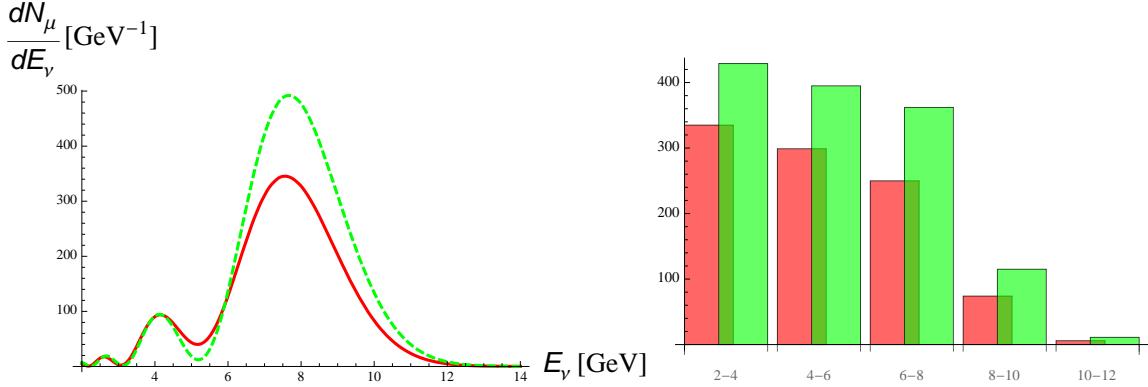


FIGURE 5. Left panel, distribution in the neutrino energy; the continuous curve (red) is for normal hierarchy, the dashed one (green) is for inverted hierarchy. Right panel, comparison of the number of events in various windows of muon energy (2-4 GeV, 4-6 GeV, etc., as indicated below the abscissa) for normal hierarchy (left bars in the back, red) and for inverted hierarchy (right bars in front, green).

large; the difference between normal and inverted hierarchy cannot be mimicked by any plausible statistical fluctuation. The signal events can be distinguished by those from atmospheric neutrinos not only because of their spectra, but also thanks to a certain degree of directionality at these energies, and, most of all, because the neutrinos arrive in bunches, due to the pulsed structure of the artificial neutrino beam.

## 6. SUMMARY AND DISCUSSION

It is widely recognized that the identification of the neutrino mass spectrum is one of the most important goals of future experimental investigations. The matter effect [5] in the Earth offers a clear way to achieve it. In this work, we argued that the ideal conditions are obtained for muon neutrinos with energies of 6 – 8 GeV propagating in the Earth for a distance of  $L = 6000 – 8000$  km that correspond to various useful possibilities. We have shown in the previous section that a (water Cherenkov) muon detector of 1 Mton along with a conventional muon neutrino beam resulting from  $10^{20}$  protons on target leads to a large signal of about 1000 signal events that are well characterized experimentally, even at the maximum distance considered above (i.e., 7800 km). The signal is composed by muons between 2 – 10 GeV that can be identified as 10 – 40 m long tracks, with 2/3 of them above 4 GeV. Due to the matter effect, the inverted hierarchy yields 30% more events than the normal hierarchy case. Thus, the difference induced by the matter effect is quite large and even a moderate understanding of the artificial neutrino beam should suffice to identify experimentally which is the neutrino mass spectrum.

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## APPENDIX A. REMARKS ON THE MATTER EFFECT

We would like to comment on the conclusions of Sect. 2 by examining the simple case of oscillations in constant matter density. With normal hierarchy, the probability that a muon converts into an electron is,

$$(10) \quad P_{\mu e} = \sin^2 \theta_{23} \sin^2 2\widetilde{\theta_{13}} \sin^2 \widetilde{\varphi} \text{ with } \widetilde{\varphi} = \frac{\widetilde{\Delta m^2} L}{4E}$$

We introduced the usual matter-modified mixing angle and squared-mass-difference

$$(11) \quad \begin{cases} \sin 2\widetilde{\theta_{13}} = \sin 2\theta_{13} / \Delta \\ \cos 2\widetilde{\theta_{13}} = (\cos 2\theta_{13} - \varepsilon) / \Delta \quad \text{where } \Delta = \pm \sqrt{(\cos 2\theta_{13} - \varepsilon)^2 + \sin^2 2\theta_{13}} \\ \widetilde{\Delta m^2} = \Delta m^2 \times \Delta \end{cases}$$

where the sign of  $\Delta$  is arbitrary and irrelevant, and the ratio between matter and vacuum term is,

$$(12) \quad \varepsilon \equiv \frac{\sqrt{2}G_F n_e}{\Delta m^2 / (2E)} \approx \frac{\rho}{5.5 \text{ g/cm}^3} \times \frac{Y_2}{1/2} \times \frac{2.4 \times 10^{-3} \text{ eV}^2}{\Delta m^2} \times \frac{E}{5.5 \text{ GeV}}$$

As usual  $G_F$  is the Fermi coupling and we identify  $\Delta m^2$  with  $\Delta m_{23}^2$ . Considering the average matter density of the Earth  $\rho = 5.5 \text{ g/cm}^3$  and  $Y_e = 1/2$ , we get  $n_e = 1.7 \times 10^{24} \text{ e}^-/\text{cm}^3$  for the electronic density. Thus, the characteristic length of MSW theory is,

$$(13) \quad L_* \equiv \frac{1}{\sqrt{2}G_F n_e} \sim 1000 \text{ km}$$

We see that the maximum of  $P_{\mu e}$  (for normal hierarchy) is obtained when  $\Delta$  is as small as possible and the phase of propagation is  $\pi/2$ , i.e., when the neutrino energy and propagation distance are,

$$(14) \quad E_{\max} = \frac{\Delta m^2 L_*}{2} \cos 2\theta_{13} \sim 5.5 \text{ GeV} \text{ and } L_{\max} = \frac{\pi L_*}{\tan 2\theta_{13}} \sim 9000 \text{ km}$$

In the case of inverted hierarchy, the matter effect depresses  $P_{\mu e}$ , that becomes negligible.

In principle, one could check this simple prediction concerning  $P_{\mu e}$ , however it is practically easier to study muons rather than electrons. Then, let us consider the survival probability  $P_{\mu\mu}$ . We want that a local maximum of  $P_{\mu\mu}$ , resulting from  $P_{\mu\tau}$  and from  $P_{\mu e}$ , is as small as possible; thus, we are interested in the case when the minimum of  $P_{\mu\tau}$  happens in the vicinity of the energy identified in Eq. 14. When the phase of oscillation of  $P_{\mu\tau}$  is close to the vacuum phase, the condition  $\Delta m^2 L / (4E_{\max}) = \pi$  gives  $L \sim 6000 \text{ km}$ . Thus, we conclude that the distance that amplifies the matter effect on  $P_{\mu\mu}$  is between 6000 and 9000 km, that supports the conclusions of the numerical analysis of Sect. 2.

Finally, we collect further technical remarks and additional clarifications concerning the matter effect. Let us write in full generality the amplitude of three flavor neutrino oscillations

$$(15) \quad \mathcal{A} = \text{Texp} \left[ -i \int dt \mathcal{H}_\nu(t) \right] = R_{23} \Delta \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \Delta^* R_{23}^t$$

where  $a_{ij}$  depend upon  $\Delta m_{23}^2$ ,  $\theta_{13}$ ,  $\Delta m_{12}^2$ ,  $\theta_{12}$ , and  $H = \pm 1$  (the type of mass hierarchy, normal/inverted): see in particular Eqs. 1, 3, 5, 6 of [12]. When the “solar”  $\Delta m_{12}^2$  is set to zero—i.e., when its effects are negligible—the only non-zero out-of-diagonal element are  $a_{13}$  and  $a_{31}$ . The probability  $P_{\ell\ell'} = |\mathcal{A}_{\ell\ell'}|^2$  becomes symmetric,  $P_{\ell\ell'} = P_{\ell'\ell}$  for each  $\ell, \ell' = e, \mu, \tau$ ; therefore, we have 3 independent probabilities and all the other ones are fixed. We can chose, e.g.,

$$(16) \quad P_{e\mu} = \sin^2 \theta_{23} |a_{13}|^2, \quad P_{e\tau} = \cos^2 \theta_{23} |a_{13}|^2, \quad P_{\mu\tau} = \sin^2 \theta_{23} \cos^2 \theta_{23} |a_{33} - a_{22}|^2,$$

so that, e.g.,  $P_{ee} = 1 - P_{\mu e} - P_{\tau e} = |a_{11}|^2$ . In the case of constant matter density considered above, the explicit solution for normal mass hierarchy reads simply:

$$(17) \quad \begin{aligned} a_{13} &= a_{31} = -i \sin \tilde{\varphi} \sin 2\tilde{\theta}_{13} \\ a_{11} &= \cos \tilde{\varphi} + i \sin \tilde{\varphi} \cos 2\tilde{\theta}_{13} = a_{33}^* \\ a_{22} &= \cos \tilde{\varphi}' + i \sin \tilde{\varphi}' \end{aligned}$$

where

$$(18) \quad \tilde{\varphi}' = \frac{\Delta m^2 L}{4E} (1 + \varepsilon)$$

Changing the sign of  $\varepsilon$ , one obtains the probability of oscillation of antineutrinos for normal hierarchy; in the approximation we are considering in this appendix, this coincides with the probability of oscillation of neutrinos for inverted hierarchy.

Eq. 10 follows from the first two equations of Eqs. 16 and 17. Then, we discuss  $P_{\mu\tau}$  close to its first minimum, that occurs for the energies where the matter effect is large. Writing

$$(19) \quad P_{\mu\tau} = \sin^2 2\theta_{23} |1 - \sqrt{P_{ee}} \exp(i\hat{\varphi})|^2 / 4$$

we see that even the relatively small  $P_{ee} \approx 0.3 - 0.4$ , due to matter effect, will not lift much  $P_{\mu\tau}$  from zero near the minimum, close to the point  $\xi = 2\pi$ . The phase  $\hat{\varphi}$ , defined by  $\sqrt{P_{ee}} \cos \hat{\varphi} = \cos \tilde{\varphi} \cos \tilde{\varphi}' - \sin \tilde{\varphi} \sin \tilde{\varphi}' \cos 2\tilde{\theta}_{13}$ , can be close to the vacuum phase: in fact, if  $\varepsilon$  is large or small in comparison to 1, then  $\cos 2\tilde{\theta}_{13} \sim +1$  and  $\tilde{\varphi} \sim \Delta m^2 L / (4E)(1 - \varepsilon)$ , so that  $\hat{\varphi} \sim \tilde{\varphi} + \tilde{\varphi}' \sim \Delta m^2 L / (2E)$ .

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